

Answers

Probability Exercises

Conversion Exercises:

- Convert the following to a fraction.
 - $0.25 = \mathbf{25/100}$
 - $0.60 = \mathbf{60/100}$
 - $0.934 = \mathbf{934/1000}$
- Convert the following from percent to probability.
 - $25\% = \mathbf{.25 \text{ probability}}$
 - $33\% = \mathbf{.33 \text{ probability}}$
 - $75\% = \mathbf{.75 \text{ probability}}$
- Convert the following from probability to percent.
 - $0.2 = \mathbf{20\%}$
 - $0.64 = \mathbf{64\%}$
 - $0.934 = \mathbf{93.4\%}$
- In flipping a coin six times, the following sequence was observed: H,T,T,T,T,T. What is the probability that on the seventh flip the coin will come up tails?

Regardless of what happened on previous flips, the probability is .50 every time the coin is flipped
- Draw one card from an ordinary deck of cards. (Express answers as fractions and decimals.)
 - What is the probability that it is the queen of hearts?

 $\mathbf{1/52 \text{ or } 1 \text{ in } 52 = .019}$

- b) What is the probability that it is either the king or the queen of hearts?

$$1/52 + 1/52 = 2/52 = 1/26 = .038$$

6. Draw two cards from an ordinary deck. What is the probability of getting both the king and queen of hearts?

For the first card drawn, there is a 2 in 52 chance of getting one of the two cards. For the second card, there is a 1 in 51 chance. The probability of drawing both the queen and king of hearts on two draws is:

$$(2/52) \times (1/51) = (2 \times 1) / (52 \times 51) = 2/2652 = 1/1326 = .00075$$

7. Draw five cards. What is the probability of drawing the A, K, Q, J, and 10 of hearts? What is the probability of drawing any royal flush?

There are 5 cards we want to choose from the deck. For the first card, there is a 5 in 52 chance (5/52) of drawing one of the cards we want. For the second card, there are 4 cards left in the 51 remaining cards in the deck, so there is a 4 in 51 chance (4/51) for the second card. Using the same method, there is a 3 in 50 chance for the third card, a 2 in 49 chance for the fourth card, and a 1 in 48 chance for the fifth card. So the probability of drawing that hand is:

$$5/52 \times 4/51 \times 3/50 \times 2/49 \times 1/48 = \\ (5 \times 4 \times 3 \times 2 \times 1) / (52 \times 51 \times 50 \times 49 \times 48) = \\ 120 / 311,875,200 = .000000384$$

This is the probability of drawing one royal flush. Since there are four possible royal flushes, one for each suit, we need to multiply this probability by 4, giving us a .00000154 probability for drawing any royal flush or approximately 1.5 in a million.

8. Two basketball teams of equal skill are involved in a four-game tournament. What is the probability of one of the teams winning the tournament in four straight games?

Each team has a one in two chance of winning each game. Therefore, the probability of winning four straight games is:

$$(1/2) \times (1/2) \times (1/2) \times (1/2) = 1/16 = .0625 \text{ or about } 6\%.$$

Challenge Level Exercises:

1. A typical roulette wheel has 38 slots that are numbered 1, 2, 3,..., 34, 35, 36, 0, and 00. The 0 and 00 slots are green. Of the remaining slots, half are red and half are black. Also, half of the integers from 1 to 36 are even and half are odd. 0 and 00 are defined as neither even or odd. A ball is rolled around the wheel and ends up in one of the slots. We assume that each slot has an equal chance.

- a) What is the probability of the ball landing in each slot?

There are 38 slots, each with an equal chance. Therefore, each slot has a 1/38 probability.

- b) What is the probability of the ball landing in a green slot? A red slot? A black slot?

2 of the 38 slots are green, so the probability of the ball landing in a green slot is 2/38 or 1/19. Of the 36 remaining slots, half are red and half are black. There are 18 red and 18 black slots. The probability of red equals the probability of black, so the probability of the ball landing in either a red or black slot is 18/38 or 9/19.

- c) What is the probability of the ball landing on an even number?

Half of the numbers from 1 to 36 are even, so there are $36/2 = 18$ even slots. Therefore, the probability of getting an even number is 18/38 or 9/19.

- d) What is the probability of getting a 1, 12, 24, or 36?

The probability for each of these numbers is 1/38, so the probability of getting one of these four is:

$$1/38 + 1/38 + 1/38 + 1/38 = 4/38 \text{ or } 2/19$$

2. In a particular lottery, 2,000,000 tickets are sold each week for \$0.50 each. Each week there are 12,009 tickets drawn and awarded prizes: 12,000 people receive \$25; 6 people win \$10,000; 2 people win \$50,000; and 1 person wins \$200,000.

- a) Determine the probability of winning each prize.

Prize	No. of Prizes Awarded	Probability of Each Prize
\$25	12,000	$12,000/2,000,000 = .006$
\$10,000	6	$6/2,000,000 = .000003$
\$50,000	2	$2/2,000,000 = .000001$
\$200,000	1	$1/2,000,000 = .0000005$

b) If you play this game, what is your “expected” payoff?

Prize x Probability

$$\begin{aligned} 25(.006) &= .15 \\ 10,000(.000003) &= .03 \\ 50,000(.000001) &= .05 \\ 200,000(.0000005) &= .10 \\ \text{Expected Winnings} &= \$0.33 \end{aligned}$$

But, don’t forget you spent 50 cents on the ticket, so the adjusted “winnings” are:

$$\$0.33 - .50 = \$-.17$$

3. Suppose you must choose between two products (A and B) to sell in your shop. Your choice depends on what the economy is going to do. If the economy goes up, you will make a profit of \$100,000 on product A or \$60,000 on product B. If the economy stays the same, you will earn a profit of \$50,000 on product A and \$40,000 on product B. If the economy goes down, you will lose \$20,000 on product A but can still earn \$10,000 on product B.

You don’t know for sure what the economy is going to do, but you might know the probabilities of these things happening. Suppose the probability of the economy going up is 0.4, the probability of it staying the same is 0.4, and the probability of it going down is 0.2.

Determine the expected profit for each product. Which product would you choose and why?

$$\begin{aligned} \text{Product A: } &.4(100,000) + .4(50,000) + .2(-20,000) = \\ &40,000 + 20,000 - 4,000 = 56,000 \end{aligned}$$

$$\begin{aligned} \text{Product B: } &.4(60,000) + .4(40,000) + .2(10,000) = \\ &24,000 + 16,000 + 2,000 = 42,000 \end{aligned}$$

The average business professional would choose Product A. However, a very conservative or cautious business professional may be willing to sacrifice some of the profit in order to avoid any risk of losing money, so he or she would choose Product B.